# Six-Dimensional Potential Energy Surface and Rovibrational Energies of the HCCN Radical in the Ground Electronic State ${ }^{\dagger}$ 

Mirjana Mladenović* and Peter Botschwina*<br>Institut für Physikalische Chemie, Universität Göttingen, Tammannstrasse 6, D-37077 Göttingen, Germany<br>Cristina Puzzarini ${ }^{\text {| }}$<br>Dipartimento di Chimica "G. Ciamician", Università di Bologna, Via Selmi 2, 40126 Bologna, Italy

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#### Abstract

We report large-scale quantum mechanical calculations for the HCCN radical in its ground electronic state. A six-dimensional potential energy surface based on MR-ACPF/cc-pVQZ ab initio energy points is developed and adjusted to reproduce experimental findings for $v_{5}^{1}$ and $v_{1}$ of HCCN. Rovibrational energy levels of HCCN and DCCN are computed for total rotational angular momentum $J=0-4$ by making use of combined (functional + point wise) coordinate representations together with contraction schemes resulting from several diagonalization/truncation steps. The classical barrier to linearity is determined to be $287 \mathrm{~cm}^{-1}$. Spectroscopic parameters are calculated for low lying states and compared with available experimental data. Energy patterns attributed to the $v_{4}$ bending mode and to the quasilinear $v_{5}$ bending mode are identified. It has been also found that $v_{2}$ and $v_{3}+\left(v_{4}^{1}, v_{5}^{1}\right)^{0,0}$ are coupled in HCCN, while the mixing between $v_{3}$ and $\left(2 v_{4}^{0}, 2 v_{5}^{0}\right)^{0,0}$ is seen in DCCN.


## 1. Introduction

Cyanocarbene, HCCN, has been subjected to extensive experimental ${ }^{1-14}$ and theoretical analysis ${ }^{15-22}$ in the last 40 years. The ground electronic state of the radical was found to have triplet multiplicity. ${ }^{1-4}$ In the early work on HCCN, great effort was put into answering the question whether the radical is linear or bent. ${ }^{1-, 5,15-19}$ A low barrier to linearity was first estimated by theoretical means. ${ }^{15-17}$ From a detailed microwave study of isotopically substituted HCCN, Brown et al. deduced in 1990 an unusually short CH bond length, what led to the conclusion that HCCN may possess a quasilinear structure. ${ }^{5}$ The $v_{5}$ fundamental transition was observed at $128.907968(40) \mathrm{cm}^{-1}$ for $\mathrm{HCCN}^{11}$ and at $2243.72708(21) \mathrm{GHz}\left(74.843 \mathrm{~cm}^{-1}\right)$ for DCCN ${ }^{13}$ by means of the far-infrared laser magnetic resonance technique. Malmquist et al. ${ }^{18}$ and Koput ${ }^{21}$ determined onedimensional optimum energy profiles along the HCC angle and employed them in conjunction with the semirigid bender approach. ${ }^{23}$ However, this model was found to be insufficient for quantitatively describing the nuclear motion in HCCN. ${ }^{18,21}$ We refer to refs 11 and 21 for an overview of the previous experimental and theoretical studies on HCCN.

In the present paper, we have undertaken a detailed fulldimensional quantum mechanical study of HCCN and DCCN in the ground electronic state. The motivation for this work is provided by the recent high-resolution spectra ${ }^{6-13}$ of HCCN and DCCN, which have established the frequencies of the $v_{5}(\mathrm{HCC})$ bending and $\nu_{1}(\mathrm{HC})$ stretching vibrations to high accuracy. The

[^0]purpose of our study is to construct a potential energy surface and to compute rovibrational energy levels. Another aim is to study the quasilinear $v_{5}$ mode under realistic conditions of the vibrating-rotating molecule and to clarify characteristic energy patterns. Our numerical results for $\mathrm{H} / \mathrm{DCCN}$ are of interest for studies on tetratomic molecules involving a hydrogen atom attached to a nearly linear chain of heavy atoms, such as, e.g., HCNO, HNCO, and HNCS.

In section 2, we address the potential energy surface (PES) constructed in the present work. The computational method used to obtain rovibrational energies for $J=0-4$ is briefly explained in section 3. Our results for HCCN and DCCN are discussed in section 4 . Section 5 contains our concluding remarks.

## 2. Potential Energy Surface

A. Construction of the PES. The six-dimensional potential energy surface for the ground $\left({ }^{3} \mathrm{~A}^{\prime \prime}\right)$ electronic state of HCCN was scanned by the multireference averaged coupled-pairfunctional (MR-ACPF) method, ${ }^{24-27}$ using the correlation consistent valence quadruple- $\zeta$ (cc-pVQZ) basis set. The Complete Active Space SCF (CASSCF) method was employed to optimize the orbitals. All calculations were performed with the MOLPRO suite of programs. ${ }^{28}$ The nuclear coordinate space was parametrized in terms of the bond-distance-bond-angle coordinates $r_{1}, r_{2}, r_{3}, \alpha, \beta$, and $\tau$, shown in Figure 1a. Here, $r_{1}=r(\mathrm{H}-\mathrm{C}), r_{2}=r(\mathrm{C}-\mathrm{C})$, and $r_{3}=r(\mathrm{C}-\mathrm{N})$. The two inplane bending angles, $\angle \mathrm{HCC}$ and $\angle \mathrm{CCN}$, are denoted respectively by $\alpha$ and $\beta$, whereas $\tau$ stands for the dihedral angle measured from the cis side. The PES was calculated for the region specified by $1.4 a_{0}<r_{1}<4.1 a_{0}, 1.9 a_{0}<r_{2}<4.6 a_{0}$, $1.7 a_{0}<r_{3}<4.3 a_{0}, 80^{\circ} \leq \alpha \leq 180^{\circ}, 110^{\circ} \leq \beta \leq 180^{\circ}$, and $0^{\circ} \leq \tau \leq 180^{\circ}$. In total, 1311 ab initio points were computed.

(a)

(b)

Figure 1. Description of the internal geometry of HCCN (a) in terms of the bond-distance-bond-angle internal coordinates $r_{1}, r_{2}, r_{3}, \alpha, \beta, \tau$ and (b) in terms of the orthogonal (diatom + diatom) internal coordinates $R, d_{1}, d_{2}, \theta_{1}, \theta_{2}$, and $\chi$.

The calculated potential energies were in the range from $-131.21429 E_{\mathrm{h}}$ to $-130.98376 E_{\mathrm{h}}$.

The ab initio points were fitted to the six-dimensional analytical expression

$$
\begin{equation*}
V=\sum_{i j k l m n} C_{i j k l m n} x_{1}^{i} x_{2}^{j} x_{3}^{k} P_{l}^{n}\left(c_{\alpha}\right) P_{m}^{n}\left(c_{\beta}\right) \cos (n \tau) \tag{1}
\end{equation*}
$$

where $c_{\alpha}=\cos \alpha, c_{\beta}=\cos \beta$, and $l, m \geq n$. For the stretching part, we use a polynomial expansion in the modified Morse coordinates $x_{\mathrm{i}}(i=1,2,3)$

$$
\begin{equation*}
x_{i}=\frac{1}{a_{i}}\left[1-\mathrm{e}^{-a_{i}\left(r_{i} / r_{i}^{\text {ref }}-1\right)}\right] \tag{2}
\end{equation*}
$$

recommended by Meyer et al. ${ }^{29}$ The angular part of $V$ is given by a multipole-like expansion, involving associated Legendre functions $P_{l}^{n}\left(c_{\alpha}\right), P_{m}^{n}\left(c_{\beta}\right)$ and Chebyshev polynomials $\cos (n \tau)$, as suggested previously. ${ }^{30}$ The functional form of eq 1 is invariant under the spatial inversion $(\tau \rightarrow 2 \pi-\tau)$ and independent from the torsion angle $\tau$ for linear arrangements $\alpha, \beta=0, \pi$.

The analytical expansion of eq 1 has six nonlinear parameters, $r_{i}^{\text {ref }}$ and $a_{i}$, which define the Morse coordinates $x_{\mathrm{i}}$ of eq 2 . In the preliminary stages of the fitting, we employed a nonlinear least-squares technique to obtain optimum values for both $r_{i}^{\text {ref }}$ and $a_{i}$. In the final fitting, $r_{i}^{\text {ref }}$ and $a_{i}$ were held constant. For given $r_{i}^{\text {ref }}$ and $a_{i}$, the expansion parameters $C_{i j k l m n}$ were determined by means of a linear least-squares procedure.

Several sets of $\{i, j, k, l, m, n\}$ were tested in our fitting of eq 1 to the ab initio points. The complete quintic expansion, augmented by the one-dimensional radial contributions up to the seventh degree and by the three-dimensional angular part of the sixth degree, was found to be sufficient for achieving a reasonable balance between the number of fitting parameters, number of input $a b$ initio points, and associated standard deviation $\sigma$. The chosen ansatz of $\{i, j, k, l, m, n\}$ defines in total 336 fitting parameters $C_{i j k l m n}$. This number of parameters was reduced by an extensive search for all significantly determinable parameters. Eliminating stepwise all parameters smaller than twice their standard deviation and most of the parameters smaller than three times their standard deviation, we arrived at a functional six-dimensional expansion consisting of 125 terms.

TABLE 1: Expansion Coefficients $C_{i j k l m n}$ (in Atomic Units) of Eq 1 for the Adjusted Six-Dimensional MR-ACPF PES Derived in This Work for the 3A" Electronic State of $\mathbf{H C C N}^{a}$

| ijklmn | $C_{i j k l m n}$ | ijklmn | $C_{i j k l m n}$ | ijklmn | $C_{i j k l m n}$ |
| :--- | ---: | :---: | ---: | :---: | ---: |
| 000000 | -0.0000836 | 000310 | -0.0016181 | 010010 | 0.1333863 |
| 200000 | 0.7423250 | 000020 | -0.4014165 | 010020 | 0.1001659 |
| 300000 | -1.2529641 | 000120 | -0.4477221 | 010100 | 0.8068241 |
| 400000 | 1.2940035 | 000220 | 0.1329813 | 010110 | 1.1190861 |
| 500000 | -1.1970301 | 000030 | -0.1959100 | 010120 | 0.3713975 |
| 600000 | 0.9368890 | 000130 | -0.2138155 | 010200 | 0.5152400 |
| 700000 | -0.4416142 | 000230 | 0.0274065 | 010210 | 0.4998266 |
| 020000 | 1.2419591 | 000040 | -0.0387080 | 010220 | 0.1619246 |
| 030000 | -2.9420632 | 000140 | -0.0431464 | 010300 | 0.0558572 |
| 040000 | 3.9165475 | 000111 | 0.0110927 | 010400 | 0.0233452 |
| 050000 | -2.4605068 | 000211 | 0.0085826 | 010111 | 0.0762339 |
| 060000 | -0.6459143 | 000311 | 0.0059613 | 010121 | -0.0115625 |
| 002000 | 1.6724940 | 000411 | -0.0010206 | 010211 | 0.0817841 |
| 003000 | -5.8652217 | 000511 | 0.0003250 | 010311 | 0.0277756 |
| 004000 | 7.8379300 | 000121 | 0.0078587 | 011010 | 0.1585303 |
| 005000 | -5.7335005 | 000221 | 0.0084313 | 011100 | -0.2973279 |
| 006000 | 2.0925619 | 000321 | 0.0033287 | 011111 | 0.0491293 |
| 007000 | -0.5721966 | 000421 | -0.0004627 | 012100 | 1.4686769 |
| 011000 | 0.8697992 | 000131 | 0.0015849 | 020010 | 0.0403994 |
| 012000 | 2.0180161 | 000231 | 0.0019083 | 020100 | -0.6353160 |
| 013000 | -2.1399322 | 000331 | 0.0009184 | 020200 | -0.5613150 |
| 014000 | -2.1759310 | 000322 | 0.0003446 | 020300 | -0.1753884 |
| 021000 | -1.1684469 | 000422 | 0.0000690 | 020111 | -0.0991483 |
| 022000 | 3.2219910 | 000232 | -0.0000945 | 021010 | 0.2756852 |
| 023000 | 3.6368392 | 001010 | 0.5458700 | 021200 | 0.3383835 |
| 031000 | -2.6150563 | 001020 | 0.5965830 | 030010 | 0.1357801 |
| 032000 | -3.3335227 | 001030 | 0.1533578 | 030100 | -0.2013531 |
| 101000 | 0.0217615 | 001100 | -0.5312928 | 030200 | -0.1253704 |
| 110000 | 0.0021936 | 001110 | -0.2864925 | 100010 | 0.1117913 |
| 120000 | -0.0357270 | 001200 | -0.3406292 | 100020 | 0.0411506 |
| 201000 | 0.0144122 | 001210 | -0.1293005 | 100100 | -0.1076529 |
| 210000 | 0.0086634 | 001300 | -0.0681403 | 100200 | -0.0385778 |
| 220000 | -0.1064960 | 001400 | -0.0189303 | 100300 | -0.0095890 |
| 000100 | -0.0868363 | 001111 | -0.0670670 | 100111 | -0.0076520 |
| 000200 | 0.1745690 | 001211 | -0.0731704 | 100211 | -0.0051029 |
| 000300 | -0.0139063 | 001311 | -0.0235973 | 101100 | 0.0284819 |
| 000400 | -0.0101806 | 001222 | 0.0016467 | 110100 | 0.0455224 |
| 000500 | -0.0043744 | 002010 | -0.7734884 | 200100 | -0.1073827 |
| 000600 | -0.0008691 | 002020 | -0.2828110 | 200200 | -0.0629688 |
| 000010 | -0.3129661 | 002100 | 0.0868854 | 200300 | -0.0198132 |
| 000110 | -0.4723798 | 002111 | 0.0366603 | 300100 | 0.0346831 |
| 000210 | 0.2439514 | 003200 | 0.1573389 |  |  |
| 0 |  | 0 |  |  |  |
| 0 |  |  |  |  |  |

${ }^{a}$ The reference distances $r_{1}^{\text {ref }}=2.01964 a_{0}, r_{2}^{\text {ref }}=2.49804 a_{0}$, and $r_{3}^{\text {ref }}=2.25743 a_{0}$ and the exponential parameters $a_{1}=0.4434, a_{2}=$ 0.8009 , and $a_{3}=0.3948$ are used for the Morse coordinates $x_{i}$ defined by eq 2 .

For the final fitting, we set a standard deviation of a $i$ th date point of energy $E_{i}$ to be $E_{i}+2000 \mathrm{~cm}^{-1}$, as previously done, e.g., by Meyer et al. ${ }^{29}$ The standard deviation of the weighted 125 -term expansion was $4.5 \mathrm{~cm}^{-1}$. This $\sigma$ is only $0.9 \mathrm{~cm}^{-1}$ larger than the $\sigma$ of $3.6 \mathrm{~cm}^{-1}$ obtained for the initial 336 -term representation. Preliminary rovibrational calculations using the 125 -term PES expansion in combination with the DVR$(+\mathrm{R})+\mathrm{FBR}$ method described below (section 3) gave the $\nu_{1}$ and $\nu_{5}$ fundamentals for HCCN, which deviate by 14.5 and 5.4 $\mathrm{cm}^{-1}$ from experiment. Our analytical representation was hence refined by adjusting the parameters $C_{200000}$ and $C_{000200}$ to available experimental data for the $\nu_{1}$ stretching ${ }^{6}$ and $\nu_{5}$ bending ${ }^{11}$ frequencies. The expansion coefficients $C_{i j k l m n}$ of the refined 125 -term PES are listed in Table 5. The original values of $C_{200000}$ and $C_{000200}$ were 0.7468463 and 0.1744197 , respectively.

For the equilibrium configuration, the MR-ACPF PES gives the harmonic frequencies of $3355,1803,1173,553,441$, and $400 \mathrm{~cm}^{-1}$ for HCCN and of $2488,1794,1134,506,431$, and $331 \mathrm{~cm}^{-1}$ for DCCN.

TABLE 2: Fundamental Transitions of HCCN and DCCN (in $\left.\mathrm{cm}^{-1}\right)^{a}$

|  | HCCN |  |  | DCCN |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | this work | expt | ref | this work | expt | ref |
| $\nu_{1}$ | 3246.66 | 3246.6573(5) | 6 | 2436.64 | 2436.3723(7) | 9 |
|  | [25.94] | [26.80] | 6 | [24.44] | [25.42] | 9 |
| $\nu_{2}$ | 1733.71 | $\sim 1750$ | 8 | 1730.27 |  |  |
|  |  | 1735 | 3 |  | 1729.5 | 3 |
|  | [49.90] |  |  | [67.84] |  |  |
| $\nu_{3}$ | 1178.57 | 1178.5 | 3 | 1150.65 | 1127 | 3 |
|  | [23.03] |  |  | [-9.86] |  |  |
| $\nu_{4}^{1}$ | 430.02 | 365(15) | 7 | 407.48 | 367(15) | 7 |
|  |  | 383(20) | 6 |  |  |  |
|  |  | 458 | 3 |  | 405 | 3 |
|  | [-20.39, -47.50] | [-21.05, -48.95] | 7 | [ $-18.36,-53.68]$ | [-19.07, -53.86] | 7 |
| $\nu_{5}^{1}$ | $128.96$ | 128.907968(40) | 11 | 75.65 | 74.845(2) | 9 |
|  |  | 145(15) | 7 |  | 90(15) | 7 |
|  | [17.88, -21.98] | [18.16, -22.40] | 7 | [8.67, -41.12] | [8.70, -41.93] | 7 |

${ }^{a}$ The number in brackets gives the difference $B_{0}-B_{v}(\mathrm{in} \mathrm{MHz})$ between the ground-state rotational constants $B_{0}$ and effective rotational constant $B_{v}$ for the vibrational state $v$. For the MR-ACPF PES, $B_{0}$ is calculated to be 0.364872 and $0.329029 \mathrm{~cm}^{-1}$ for HCCN and DCCN, respectively. The corresponding experimental values due to McCarthy et al. ${ }^{7}$ are 0.366465 and $0.330441 \mathrm{~cm}^{-1}$ for HCCN and DCCN, respectively.

TABLE 3: Transitions Involving the Bending $v_{5}$ and $v_{4}$ Vibrations ${ }^{a}$

|  | HCCN |  |  | DCCN |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | this work | expt | ref | this work | expt | ref |
| $2 \nu_{5}^{2}$ | 343.44 | 341.73 | 12 | 210.89 | 207.95 | 12 |
|  | [-27.53] | $\begin{aligned} & 340(15) \\ & {[-1.01]} \end{aligned}$ | 7 | [-32.03] | $\begin{aligned} & 215(15) \\ & {[-30.71]} \end{aligned}$ | 7 |
| $2 \nu_{5}^{0}$ | 460.97 | 435(20) | 7 | 317.92 | 311(15) | 7 |
|  | [-12.86] | [-16.60] | 7 | [-15.91] | [-18.40] | 7 |
| $\left(v_{4}^{1}, v_{5}^{1}\right)^{0,1}$ | 564.77 | 525(20) | 7 | 502.35 |  |  |
|  | [-34.12] | [-35.84] | 7 | [-40.35] |  |  |
| $\left(v_{4}^{1}, v_{5}^{1}\right)^{0,0}$ | $575.81$ | 540(20) | 7 | $516.77$ |  |  |
|  | [-26.59] | [-25.55] | 7 | [-107.8] |  |  |
| $\begin{aligned} & \left(v_{4}^{1}, v_{5}^{1}\right)^{2, p} \\ & 3 v_{5}^{3} \end{aligned}$ | 560.10 |  |  | 495.97 |  |  |
|  | 618.23 | 625(20) | 7 | 386.84 | 400(20) | 7 |
|  | [-44.79] | [-2.53] | 7 | [-43.69] | [-43.12] | 7 |
| $3 \nu_{5}^{1}$ | $725.75$ |  |  | $514.90$ |  |  |
|  | $[-9.43,-38.50]$ |  |  | $[56.56,-53.20]$ |  |  |
| $\begin{aligned} & 4 v_{5}^{4} \\ & 4 v_{5}^{2} \end{aligned}$ | 939.71 |  |  | $593.89$ |  |  |
|  | 1007.17 |  |  | $697.41$ |  |  |
|  | [-45.00] |  |  | [-96.43] |  |  |
| $4 \nu_{5}^{0}$ | 1088.61 |  |  | 744.02 |  |  |
|  | [-37.83] |  |  | [-58.15] |  |  |
|  | 851.09 |  |  | 837.76 |  |  |
| $2 v_{4}^{2}$ | 860.41 |  |  | 821.85 |  |  |
| $3 v_{4}^{1}$ | 1273.46 |  |  | 1261.92 |  |  |
| $3 \nu_{4}^{3}$ | 1290.79 |  |  | 1240.88 |  |  |
| $4 \nu_{4}^{8}$ | $1686.79$ |  |  | 1689.92 |  |  |
| $4 \nu_{4}^{2}$ | $1695.14$ |  |  | 1681.57 |  |  |
| $4 v_{4}^{4}$ | 1721.89 |  |  | 1663.58 |  |  |

${ }^{a}$ For more details, see Table 2.
B. Topography of the PES. The two-dimensional maps of the adjusted six-dimensional MR-ACPF PES are displayed in Figure 2. These are made by freezing the remaining four coordinates at their equilibrium values. The contour plots involving $\{\alpha, \tau\}$ and $\{\beta, \tau\}$ (the second and third plot in the second row of Figure 2) clearly show that HCCN is a planar molecule possessing only a trans minimum. The geometrical parameters of the PES minimum are given by $r_{1}^{\mathrm{e}}=2.020 a_{0}$, $r_{2}^{\mathrm{e}}=2.499 a_{0}, r_{3}^{\mathrm{e}}=2.257 a_{0}, \alpha^{\mathrm{e}}=146.6^{\circ}, \beta^{\mathrm{e}}=174.7^{\circ}$, and $\tau^{\mathrm{e}}=180^{\circ}$. As seen in Figure 2, the coupling between the HCC bending angle $\alpha$ and the CC bond length $r_{2}$ is especially prominent. Figure 2 also shows the variation of $r_{1}, r_{2}, r_{3}, \beta, \tau$ along the minimum energy path (MEP) in the direction of $\alpha$, as well the variation of $\tau$ along the MEP for $\beta$. The MEP along a chosen coordinate is computed by minimizing $V$ with respect
to the remaining five coordinates. Along the MEP for $\alpha$, the optimum triple ( $r_{1}, r_{2}, r_{3}$ ) assumes the value of $\left(2.042 a_{0}, 2.565 a_{0}\right.$, $\left.2.236 a_{0}\right)$ for $\alpha=120^{\circ}$ and of $\left(2.009 a_{0}, 2.447 a_{0}, 2.281 a_{0}\right)$ for $\alpha=180^{\circ}$, yielding changes of $-0.033 a_{0},-0.118 a_{0}$, and $+0.045 a_{0}$ for $r_{1}, r_{2}$, and $r_{3}$, respectively. Malmquist et al. ${ }^{18}$ and Koput $^{21}$ also found that the straightening of the HCC bending angle decreases $r_{2}(\mathrm{CC})$ and increases $r_{3}(\mathrm{CN})$ in HCCN.

The variation of the potential energy along the MEP for $\alpha$, $\beta$, and $\tau$ is displayed in Figure 3. The barrier to linearity for HCC and CCN is calculated to be 287 and $84 \mathrm{~cm}^{-1}$ along the respective MEPs. The torsion barrier, i.e., the energy difference between the optimum cis and trans HCCN conformations, is $84 \mathrm{~cm}^{-1}$. In Figure 3, we also show the one-dimensional cut, $V^{\text {cut }}$, of the PES along the angle $\alpha$, which is obtained by freezing the other five coordinates at their equilibrium values. On $V^{\text {cut }}$,

TABLE 4: Combination Bands Involving the Stretching $v_{1}$ Vibration ${ }^{a}$

|  | HCCN |  |  | DCCN |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | this work | expt | ref | this work | expt | ref |
| $\nu_{1}+v_{5}^{1}$ | 3356.40 | 3355.504(2) | 10 | 2500.26 | 2499.572(1) | 9 |
|  | [43.73, 1.70] | [45.33, 2.76] | 10 | [37.18, -16.53] | [32.72, -19.11] | 9 |
| $\begin{aligned} & v_{1}+v_{5}^{1}-v_{5}^{1} \\ & v_{1}+2 v_{5}^{0} \end{aligned}$ | 3227.44 | 3226.597(2) | 6 | 2424.60 | 2424.727(1) | 9 |
|  | 3690.06 |  |  | 2750.61 |  |  |
|  | [7.48] |  |  | [11.85] | [12.29] | 9 |
| $\nu_{1}+2 \nu_{5}^{0}-2 \nu_{5}^{0}$ | 3229.10 |  |  | 2432.68 | 2432.1316(11) | 9 |
| $\nu_{1}+2 \nu_{5}^{2}$ | 3552.69 |  |  | 2623.67 |  |  |
|  | [-2.68] | [24.68] | 12 | [-8.74] | [-5.07] | 9 |
| $\nu_{1}+2 \nu_{5}^{2}-\nu_{5}^{1}$ | 3423.73 | 3420.6663(6) | 12 | 2546.71 | 2544.7432(9) | 12 |
| $\nu_{1}+3 v_{5}^{3}$ | 3809.80 |  |  | 2788.33 |  |  |
| $\nu_{1}+3 \nu_{5}^{3}-2 \nu_{5}^{2}$ | 3464.17 | $3460.9116(8)$ | 12 | $2575.46$ | 2573.4795(7) | 12 |
| $\nu_{1}+2 \nu_{5}^{2}-2 \nu_{5}^{2}$ | 3209.25 | 3207.8482(6) | 12 | $2412.78$ | 2411.636(3) | 9 |
| $\nu_{1}+\nu_{4}^{1}$ | 3673.97 |  |  | $2836.22$ |  |  |
|  | [5.41, -22.63] | [8.76, -23.32] | 6 | [9.03, -36.38] |  |  |
| $\nu_{1}+\nu_{4}^{1}-\nu_{4}^{1}$ | 3243.95 | 3244.084(1) | 6 | 2428.74 |  |  |
| $\nu_{1}+\nu_{5}^{1}-\nu_{1}$ | 109.01 | 108.85 | 12 | 62.96 | 63.20 | 12 |
| $\nu_{1}+2 \nu_{5}^{2}-\nu_{1}$ | 303.84 | 302.92 | 12 | 185.07 | $183.22$ | 12 |
| $\nu_{1}+3 \nu_{5}^{3}-\nu_{1}$ | 558.77 | 555.98 | 12 | 347.76 | 345.06 | 12 |
| $\nu_{1}+2 \nu_{5}^{0}-\nu_{1}$ | 443.40 |  |  | 313.97 |  |  |

${ }^{a}$ For more details, see Table 2.

TABLE 5: Bending Frequencies $\boldsymbol{v}_{5}^{\text {eff }}\left(\right.$ in $\mathrm{cm}^{-1}$ ) of HCCN and DCCN for the Ground-State (gs), $v_{1}, v_{2}$, and $v_{3}{ }^{a}$

|  | HCCN |  |  |  | DCCN |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\nu_{5}^{\text {eff }}$ | $V_{\text {eff }}^{\text {lin }}$ | $\theta_{1}^{\min }$ |  | $\nu_{5}^{\text {eff }}$ | $V_{\text {eff }}^{\text {lin }}$ |  |
| gs | 129 | 257 | 148 |  | $\theta_{1}^{\text {min }}$ |  |  |
| $\nu_{2}$ | 137 | 179 | 150 | 92 | 274 | 150 |  |
| $\nu_{3}$ | 116 | 323 | 147 | 59 | 333 | 152 |  |
| $\nu_{1}$ | 109 | 332 | 146 | 63 | 346 | 149 |  |
|  |  |  |  |  |  |  |  |

${ }^{a}$ The height of the barrier to linearity $V_{\mathrm{eff}}^{\mathrm{lin}}\left(\right.$ in $\left.\mathrm{cm}^{-1}\right)$ and the location of the minimum $\theta_{1}^{\min }$ (in deg) are additionally shown.
the arrangement with $\alpha=180^{\circ}$ is lying $528 \mathrm{~cm}^{-1}$ above the absolute minimum and is thus $241 \mathrm{~cm}^{-1}$ higher than the corresponding MEP value. A similar one-dimensional cut of the PES in the direction of $\beta$ shows no noticeable deviation from the corresponding MEP.

## 3. Method for Rovibrational Calculations

In the bound state variational calculations for the rovibrational energies of H/DCCN, we have employed the orthogonal (diatom + diatom) internal vectors $\mathbf{R}, \mathbf{d}_{1}, \mathbf{d}_{2}$ in the body-fixed formulation. ${ }^{31}$ The vectors $\mathbf{d}_{1}$ and $\mathbf{d}_{2}$ are chosen to be the HC and CN bond distance vectors, whereas $\mathbf{R}$ is the vector joining the centers of mass of the HC and CN subunits. The internal vectors are parametrized in terms of the three distances $R, d_{1}$, and $d_{2}$, the two bending angles $\theta_{1}$ and $\theta_{2}$, and the dihedral angle $\chi$, schematically shown in Figure 1b. The reduced masses $\mu_{R}, \mu_{d_{1}}$, and $\mu_{d_{2}}$ associated with $\mathbf{R}, \mathbf{d}_{1}$, and $\mathbf{d}_{2}$ are given for HCCN explicitly by $\mu_{d_{1}}=m_{\mathrm{H}} m_{\mathrm{C}} /\left(m_{\mathrm{H}}+m_{\mathrm{C}}\right), \mu_{d_{2}}=m_{\mathrm{C}} m_{\mathrm{N}} /\left(m_{\mathrm{C}}+m_{\mathrm{N}}\right)$, and $\mu_{R}=m_{\mathrm{HC}} m_{\mathrm{CN}} / M$, where $m_{\mathrm{A}}$ stands for the mass of the unit A and $M$ for the overall mass of HCCN. The two-vector embedded body-fixed (BF) reference frame is chosen such that the $z$-axis of the BF is aligned with the vector $\mathbf{R}$ and the $z \wedge x$ BF plane is the plane spanned by $\mathbf{R}$ and $\mathbf{d}_{1}$.

The MR-ACPF PES minimum in terms of the (diatom + diatom) coordinates is given by $R^{\mathrm{e}}=3.845 a_{0}, \theta_{1}^{\mathrm{e}}=149.6^{\circ}$, $\theta_{2}^{\mathrm{e}}=2.31^{\circ}$, and $\chi^{\mathrm{e}}=180^{\circ}$, while $d_{1}^{\mathrm{e}}=r_{1}^{\mathrm{e}}$ and $d_{2}^{\mathrm{e}}=r_{3}^{\mathrm{e}}$.

For an entirely orthogonal set of the internal vectors, such as the (diatom + diatom) vectors of Figure 1b, the body-fixed
expression of the kinetic energy operator $\hat{T}$ from ref 31 can be rewritten as a sum of several contributions

$$
\begin{equation*}
\hat{T}=\hat{T}_{\mathrm{vib}}+\hat{T}_{\mathrm{rot}}+\hat{T}_{\mathrm{cv}}+\hat{T}_{\mathrm{cr}} \tag{3}
\end{equation*}
$$

where $\hat{T}_{\text {vib }}$ and $\hat{T}_{\text {rot }}$ stand for the vibrational and rotational parts. The term $\hat{T}_{\mathrm{cv}}$ is Coriolis-like kinetic coupling in the vibrational angular subspace, whereas $\hat{T}_{\text {cr }}$ is Coriolis-like kinetic coupling between the rotation and vibrational angular motion. The four contributions are explicitly given by

$$
\begin{equation*}
\hat{T}_{\mathrm{vib}}=\hat{T}_{\mathrm{str}}+\hat{T}_{\text {bend }} \tag{4}
\end{equation*}
$$

$$
\begin{align*}
& 2 \hat{T}_{\text {bend }}=f\left(R, d_{2}\right) \hat{l}_{2}^{2} \\
& \quad-f\left(R, d_{1}\right)\left[\hbar^{2}\left(\partial^{2} \theta_{1}+\cot \theta_{1} \partial_{\theta_{1}}\right)-\left(\hat{J}_{z}-\hat{l}_{2 z}\right)^{2} / \sin ^{2} \theta_{1}\right]  \tag{5}\\
& \begin{aligned}
2 \hat{T}_{\text {rot }} & =f_{R}\left[\hat{J}^{2}-2\left(\hat{J}_{z}-\hat{l}_{2 z}\right)^{2}\right] \\
\hat{T}_{\text {cv }} & =f_{R}\left(-i \hbar \hat{l}_{2 y} \partial_{\theta_{1}}+\cot \theta_{1} \hat{l}_{2 x} \hat{l}_{2 z}\right) \\
\hat{T}_{\text {cr }} & =f_{R}\left[i \hbar \hat{J}_{y} \partial_{\theta_{1}}-\hat{J}_{x} \hat{l}_{2 x}-\hat{J}_{y} \hat{l}_{2 y}-\hat{J}_{z} \hat{l}_{2 z}\right. \\
& \left.\quad+\cot \theta_{1}\left(\hat{J}_{x} \hat{J}_{z}-\hat{J}_{x} \hat{l}_{2 z}-\hat{l}_{2 x} \hat{J}_{z}\right)\right]
\end{aligned} \tag{6}
\end{align*}
$$

where $\hat{J}$ is the angular momentum of the nuclear frame and $\hat{T}_{\mathrm{str}}=\hat{T}_{\mathrm{str}}(R)+\hat{T}_{\mathrm{str}}\left(d_{1}\right)+\hat{T}_{\mathrm{str}}\left(d_{2}\right)$ stands for the kinetic energy of the three stretching vibrations. The term $\hat{T}_{\mathrm{cv}}$ affects only vibrational energies and may lead to vibrational l-type doubling/ resonance effects. Rotational $l$-type doubling/resonance effects may arise from $\hat{T}_{\text {cr }}$.

In eqs $4-7, \partial_{\theta_{1}}$ stands for $\partial / \partial \theta_{1}, \partial^{2} \theta_{1}$ for $\partial^{2} / \partial \theta_{1}^{2}$, and $f_{R}$ for $1 / \mu_{R} R^{2}$. The function $f\left(R, d_{i}\right)$, defined as $1 / \mu_{R} R^{2}+1 / \mu_{d_{i}} d_{i}^{2}$, is the inverse of the reduced mass associated with the bending coordinate $\theta_{i}$. The vibrational angular momentum operator associated with $\mathbf{d}_{2}$ is $\hat{\mathbf{l}}_{2}=\hat{\mathbf{l}}_{2}\left(\theta_{2}, \chi\right)$. The body-fixed projections of $\hat{J}$ are $\hat{J}_{x}, \hat{J}_{y}, \hat{J}_{z}$.

The standard rotation-angular basis in the body-fixed formulation is given for tetratomic molecules by the following product ${ }^{31}$

$$
\begin{equation*}
\bar{P}_{l_{1}}^{|k-K|}\left(\cos \theta_{1}\right) Y_{l_{2}}^{k}\left(\cos \theta_{2}, \chi\right)|J K M\rangle \tag{9}
\end{equation*}
$$



Figure 2. Two-dimensional contour plots of the six-dimensional MR-ACPF PES, obtained by freezing the remaining four coordinates at their equilibrium values. Contour lines are drawn in intervals of $250 \mathrm{~cm}^{-1}$ with the first contour placed at $50 \mathrm{~cm}^{-1}$. The dashed lines show the variation of the internal coordinate shown on the $y$-axis along the minimum energy path for the coordinate displayed on the $x$-axis.


Figure 3. Minimum energy path (solid line) along the HCC angle $\alpha$, the CCN angle $\beta$, and the torsion angle $\tau$. The one-dimensional cut $V^{\text {cut along }}$ $\alpha$ is given by the dashed line.
where $\bar{P}_{l}^{k}(\cos \theta)$ are normalized associated Legendre functions and $Y_{l_{2}}^{k}\left(\cos \theta_{2}, \chi\right)$ are spherical harmonics with the CondonShortley phase convention. ${ }^{32}$ The quantum number of the projection of $\hat{\mathbf{l}}_{2}$ onto the $z$-axis of the BF is $k$. The symmetric top eigenfunctions ${ }^{33}$ are denoted by $|J K M\rangle$, where $M$ and $K$ are the quantum numbers for the space-fixed Z-projection and the body-fixed $z$-projection of $\hat{\mathbf{J}}$, respectively. The action of the kinetic energy operator $\hat{T}$ of eq 3 on the basis functions of eq 9 can be solved analytically. ${ }^{31}$

The total rotational angular momentum $J$, its space-fixed projection $M$, and the parity $p$ are strictly conserved quantum numbers for the eigenstates of a tetratomic molecule. For a given $J$, a parity-adapted angular basis can thus facilitate the computation. A detailed description of the parity-adapted rotation-angular basis functions employed in the present work can be found in ref 31 .

In actual computations, the discrete variable representation (DVR) for the bending angles $\theta_{1}$ and $\theta_{2}$ is used together with the finite basis representation (FBR) of the dihedral angle $\chi$, the eigenfunction basis in $d_{1}, d_{2}$, and the discretized Jacobi coordinate $R$. The DVR+FBR is combined with a sequential diagonalization/truncation scheme. This approach is termed $\mathrm{DVR}(+\mathrm{R})+\mathrm{FBR}^{34}$ and involves no dynamical approximation. In the $\operatorname{DVR}(+\mathrm{R})+\mathrm{FBR}$ method, contributions to the rovibra-
tional Hamiltonian are taken sequentially into account. After the inclusion of new term(s), the higher-dimensional Hamiltonian matrix is calculated and diagonalized, while the basis set is truncated by retaining only a preselected number of eigenvectors for subsequent computations. For chosen $K, k, R_{z}$ and parity $p$, where $R_{z}$ stands for a discrete point in $R$, the terms $\hat{T}_{\text {bend }}+\hat{T}_{\text {str }}\left(d_{1}\right)+\hat{T}_{\text {str }}\left(d_{2}\right)$, and $\hat{T}_{\text {rot }}$ are first considered together with the potential energy contribution diagonal in $k$. In the next step, the term $\hat{T}_{\mathrm{cv}}$ is added along with the potential energy contribution nondiagonal in $k$, resulting in five-dimensional eigenvalue problems for each of the pairs $\left(K, R_{z}\right)$. The inclusion of the kinetic energy in $R$ leads to the formulation of six-mode eigenvalue problems for a fixed $K$. The final nine-dimensional rovibrational Hamiltonian matrix is constructed by including the term $\hat{T}_{\text {cr }}$ and is of relatively modest size owing to a very efficient contraction scheme of the $\operatorname{DVR}(+\mathrm{R})+$ FBR approach. At different stages of calculation, the Hamiltonian matrices are diagonalized by conventional dense matrix algorithms to give both eigenvalues and eigenfunctions.
A. Adiabatic Projection Scheme. The computational strategy of the $\operatorname{DVR}(+\mathrm{R})+\mathrm{FBR}$ approach readily permits the construction of the adiabatic (zero-order) basis, consisting of eigenvectors computed in the adiabatic $R$-stretch approximation.


Figure 4. Low-lying states of HCCN calculated for total rotational angular momentum $J$ and parity $p$. The quantum number assignments are made according to the linear molecular model. The quantum labels of the states denoted by $a, b$, and $c$ are $3 v_{5}^{1},\left(v_{4}^{1}, 2 v_{5}^{2}\right)^{1, \mathrm{p}}$, and $\left(v_{4}^{1}, 2 v_{5}^{0}\right)^{1, \mathrm{p}}$, respectively. For the states shown, the quantum label $K$ for the projection of $\hat{\mathbf{J}}$ on the body-fixed $z$-axis is essentially a good quantum number.

The five-mode eigenstates calculated for a grid of the discrete points $R_{z}$ form adiabatic potentials $V_{\text {adi }}^{i}$ for $R$. The stretching states $|\alpha\rangle$, adiabatically separated from the remaining five vibrational modes, can be obtained for the $i$ th five-mode state by employing the adiabatic Hamiltonian $\hat{H}_{\mathrm{adi}}^{i}=\hat{H}_{\mathrm{adi}}^{i}(R)$, consisting of the kinetic energy $\hat{T}_{\text {str }}=\hat{T}_{\text {str }}(R)$ and the adiabatic potential $V_{\text {adi }}^{i}$. To monitor the evaluation of the adiabatic representation within the $\mathrm{DVR}(+\mathrm{R})+\mathrm{FBR}$ computational scheme, the quantum state character correlation scheme is used. The latter scheme was originally developed for the adiabatic torsion approximation and the $\operatorname{DVR}(6)$ approach. ${ }^{35}$

The adiabatic vectors $\mid \alpha, i)=|\alpha\rangle|i\rangle$ are used in the DVR$(+\mathrm{R})+\mathrm{FBR}$ for the calculation of adiabatic expansions of numerically exact wave functions $|n\rangle$

$$
\begin{equation*}
\left.|n\rangle=\sum_{\alpha, i} C_{\alpha, i}{ }^{n} \mid \alpha, i\right) \tag{10}
\end{equation*}
$$

The latter expansion supplies a zero-order origin of $|n\rangle$ by locating the dominant adiabatic contribution among all $\mid \alpha, i$, , providing, thus, an important foundation for quantum number assignments. The adiabatic projection of eq 10 is easily extended to include also the $J \neq 0$ situation.
B. Numerical Details. The numerical parameters used to compute the rovibrational energy levels of H/DCCN for $J=$ $0-4$ by means of the $\operatorname{DVR}(+\mathrm{R})+\mathrm{FBR}$ are summarized here. The pointwise representation of the Jacobi distance $R$ is obtained in the spirit of the potential optimized DVR method of Echave and Clary. ${ }^{36}$ For HCCN, we used 12 discrete $R_{z}$ points distributed nonevenly between $3.54 a_{0}$ and $4.43 a_{0}$. Three eigenfunctions for $d_{1}$ and five eigenfunctions for $d_{2}$ were constructed for the reference potential determined by the equilibrium $\mathrm{H} / \mathrm{DCCN}$ geometry. The maximum value $k^{\text {max }}$ for the projection quantum number $k$ was 5 in $J=0$ and 9 in $J=4$ calculations. The integrals over the dihedral angle $\chi$ were solved by means of Gauss-Chebyshev quadrature of order 11. For the bending angles, Gauss-Legendre DVR points were used, determined from $P_{l_{\text {max }}}^{k}$ of $k=0,1, \cdots, k^{\max }$ with $l_{1}^{\max }$ of 35 for $\theta_{1}$ and $l_{2}^{\max }$ of 95 for $\theta_{2}$. The Gauss-Legendre DVR set was truncated, keeping only the points distributed between 90 and $180^{\circ}$ for $\theta_{1}$ and between 0 and $40^{\circ}$ for $\theta_{2}$. The primary basis included 347400 functions for $J=0$ and 3495960 for $J=4$. The truncation of the basis set was made with the help of the cutoff parameters
$N_{k}{ }^{\text {max }}, N_{z}{ }^{\text {max }}$, and $N_{K}{ }^{\text {max }}$, which give the number of eigenvectors to be saved after the diagonalization of the ( $K, R_{z}, k$ )-blocks, ( $K, R_{z}$ )-blocks, and $K$-blocks, respectively. The parameters $N_{k}{ }^{\text {max }}$, $N_{R}{ }^{\text {max }}, N_{K}{ }^{\max }$ were chosen to be 150,800 , and 500 , respectively. The size of the six-dimensional matrices of the $K$-blocks was thus 9600 . The size of the final rovibrational Hamiltonian matrix was $1000,1500,2000$, and 2500 for $J=1,2,3,4$, respectively.

## 4. Results

Selected results obtained for $J=0-4$ by means of the DVR$(+\mathrm{R})+\mathrm{FBR}$ method and the MR-ACPF PES are shown in Tables $2-5$ and Figures 4-7. In these tables, the level energies are given relative to the ground vibrational state, calculated to be $3787.357 \mathrm{~cm}^{-1}$ for HCCN and $3289.913 \mathrm{~cm}^{-1}$ for DCCN.
A. Quantum State Labels. Following the conventional linear molecule notation, the state assignments can be made in terms of $v_{1}, v_{2}, v_{3}, v_{4}^{1_{4}}$, and $v_{5}^{1_{5}}$. The mode labels $v_{1}, v_{2}, v_{3}, v_{4}$, and $v_{5}$ refer respectively to CH stretch, CN stretch, CC stretch, CCN bend, and HCC bend, whereas $l_{4}, l_{5}$ denote the vibrational angular momentum quantum numbers. The permitted values of $l_{i}$ are $v_{i}, v_{i}-2, v_{i}-4, \ldots,-v_{i}$ for $i=4,5$.

Since $K, k \geq 0$ holds in the parity-adapted formulation, ${ }^{31}$ we employ in the present work the quantum label $\left(v_{4} v_{4}^{|l 4|}\right.$, $\left.v_{5} v_{5}^{|l| l \mid}\right)^{K, p}$, where $p$ stands for parity ( $p=0$ for even parity and $p=1$ for odd parity). In the body-fixed formulation used here, $l_{5}$ correlates to the projection quantum number $k$. Angular momentum conservation additionally restricts $l_{4}, l_{5}$ to $l_{4}+l_{5}=$ $l=K \leq J$, as seen in eq 9 and ref 31 . For quantum states given by $v_{4}=0$, we use the label $v_{5} v_{5}^{K}$, and similar for the $v_{5}=0$ situation.

The low-lying rovibrational states of HCCN calculated for $J=0,1$, and 2 and both parities are found in Figure 4. One may note that the levels $\nu_{4}^{1}$ and $v_{5}^{1}$ occur only for $J \geq 1$, whereas the states $2 v_{5}^{2}, 2 v_{4}^{2}$, and $\left(v_{4}^{1}, v_{5}^{1}\right)^{2, p}$ become accessible for $J \geq 2$. The two $K=0$ levels shown by the dashed lines in Figure 4 are assigned $\left(v_{4}^{1}, v_{5}^{1}\right)^{0, \mathrm{p}}$ in the notation used here or $v_{4}^{ \pm 1}+v_{5}^{\mp 1}$. The alternative assignment for the two $K=2$ levels $\left(v_{4}^{1}, v_{5}^{1}\right)^{2, \mathrm{p}}$ of Figure 4 is $v_{4}^{ \pm 1}+v_{5}^{ \pm 1}$.

In the bent molecule notation, the levels $2 v_{5}^{0},\left(v_{4}^{1}, v_{5}^{1}\right),{ }^{0,0}$ and $\left(\nu_{4}^{1}, v_{5}^{1}\right)^{0,1}$ are respectively $\nu_{5}^{\mathrm{b}}, \nu_{4}^{\mathrm{b}}$, and $\nu_{6}^{\mathrm{b}}$, whereas $\nu_{5}^{1}$ and $2 \nu_{5}^{2}$ correlate with the $K=1$ and $K=2$ ( $a$-type) transitions from


Figure 5. Wave function probability amplitudes for the ground vibrational state (gs), $2 v_{5}^{0},\left(v_{4}^{1}, v_{5}^{1}\right),{ }^{0,0}$ and $2 v_{4}^{0}$ of HCCN for $J=0$. The remaining four coordinates are kept constant at their equilibrium values. The contours seen in the $\left(\theta_{1}, \theta_{2}\right)$ plane show where the probability amplitude takes $10 \%, 20 \%, 30 \%, 40 \%, 50 \%, 60 \%, 70 \%, 80 \%, 90 \%$ of its maximum value.
the ground state. The levels assigned $v_{4}^{1}$ in Figure 4 are, however, not easy to interpret by means of the bent molecular model.

The two-dimensional $\left(\theta_{1}, \theta_{2}\right)$ cuts of the wave function probability amplitudes for the ground vibrational state, $2 v_{5}^{0}$, $\left(v_{4}^{1}, v_{5}^{1}\right),{ }^{0,0}$ and $2 v_{4}^{0}$ of HCCN are shown in Figure 5. The welldefined nodal patterns seen for $2 v_{5}^{0}$ and $2 v_{4}^{0}$ in Figure 5 clearly imply the excitation of the $\theta_{1}$ bending motion in $2 v_{5}^{0}$ and the excitation of the $\theta_{2}$ bending motion in $2 v_{4}^{0}$, confirming the linear molecule convention for HCCN.

A striking feature in Figure 5 is a large wave function amplitude seen at the linear geometry, $\theta_{1}=\pi$, even in the ground vibrational state. This property is also visible in Figure 6 , where the wave function probability amplitude integrated over $R, d_{1}, d_{3}, \theta_{2}, \chi$ and three Euler angles are shown for the ground vibrational state, $v_{5}^{1}, 2 v_{5}^{0}, 3 v_{5}^{1}$, and $4 v_{5}^{0}$. Although the ground vibrational state at $3787 \mathrm{~cm}^{-1}$ lies $\approx 90 \mathrm{~cm}^{-1}$ below the barrier to linearity, the ground-state wave function amplitude differs significantly from zero at $\theta_{1}=\pi$. For the levels with $l_{5}=1$, the wave function amplitude at $\theta_{1}=\pi$ is strictly zero as a consequence of the rotational symmetry, Figure 6 and eq 9. The states with $l_{5} \neq 0$ exceed the barrier to linearity.

The vibrationally averaged bending amplitude $\left\langle\theta_{i}-\theta_{i}{ }^{\mathrm{e}}\right\rangle$ is calculated as $\arccos \left[\left\langle\cos \left(\theta_{i}-\theta_{i}{ }^{e}\right)\right\rangle\right]$. For the ground vibrational state, $2 v_{5}^{0},\left(v_{4}^{1}, v_{5}^{1}\right),{ }^{0,0}$ and $2 v_{4}^{0}$ of Figure 5, the amplitude $\left\langle\theta_{1}-\right.$ $\left.\theta_{1}^{\mathrm{e}}\right\rangle$ describing the quasilinear mode is calculated to be 11.4,
$16.0,10.6$, and $12.0^{\circ}$, respectively. For $\left\langle\theta_{2}-\theta_{2}^{\mathrm{e}}\right\rangle$, we obtained $4.2,4.2,6.3$, and $8.0^{\circ}$.
B. Fundamental Transitions. The fundamental transitions of HCCN and DCCN are summarized in Table 2. The MRACPF PES, adjusted to reproduce experimental wavenumbers for $v_{5}^{1}$ and $v_{1}$ of HCCN (section 2), provides a good description of $\nu_{5}$ and $\nu_{1}$ also for DCCN. As seen in Table 2, the $\nu_{1}$ and $\nu_{5}$ frequencies calculated for DCCN differ by +0.27 and +0.81 $\mathrm{cm}^{-1}$ from the high-resolution spectroscopic results of Sun et al. ${ }^{9}$

So far, no high-resolution spectroscopic study has been reported for the $v_{3}$ and $v_{4}$ vibrations of either HCCN or DCCN. Dendramis and Leroi ${ }^{3}$ derived from matrix isolation infrared spectra five fundamental transitions for various isotopically substituted forms. McCarthy et al. ${ }^{7}$ and Morter et al. ${ }^{6}$ estimated the transition energies of $\nu_{4}$ and $v_{5}$ from relative intensity measurements in the microwave and infrared region, respectively.

Miller et al. ${ }^{8}$ recorded the high-resolution infrared spectrum of HCCN in the region of the $\nu_{2}$ stretching fundamental and found six separate vibrational bands. However, no vibrational identification could be made. Fermi resonance between $\nu_{2}$ and $v_{3}+\left(v_{4}^{1}, v_{5}^{1}\right)^{0,0}$ was suggested in ref 8 as a probable intensity borrowing mechanism in the region of $v_{2}$.

In the present work, the energy spectra calculated for HCCN and DCCN are studied in great detail with the help of the adiabatic expansions of eq 10 . This analysis led to the


Figure 6. Wave function probability amplitudes integrated over the five internal coordinates $R, d_{1}, d_{2}, \theta_{2}, \chi$ and three Euler angles for the ground vibrational state $(g s), v_{5}^{1}, 2 v_{5}^{0}, 3 v_{5}^{1}$, and $4 v_{5}^{0}$ of HCCN. The levels shown are calculated for $J=1$ and $p=1$. The effective groundstate potential energy profile along $\theta_{1}$ is denoted by $V_{\text {eff }}{ }^{0}$. The horizontal lines indicate the level energy.
identification of several resonances involving the fundamental stretching $v_{2}$ and $v_{3}$ vibrations. For HCCN, we have

$$
\begin{gather*}
\left.\left.\left|v_{2}\right\rangle \approx 0.799 \mid 0,9\right)-0.439 \mid 1,2\right)  \tag{11}\\
\left.\left.\left|v_{3}+\left(v_{4}^{1}, v_{5}^{1}\right)^{0,0}\right\rangle \approx 0.534 \mid 0,9\right)+0.725 \mid 1,2\right)
\end{gather*}
$$

and

$$
\begin{gather*}
\left.\left.\left|v_{3}\right\rangle \approx-0.428 \mid 0,5\right)+0.818 \mid 1,0\right)  \tag{12}\\
\left.\left.\left|\left(v_{4}^{1}, 3 v_{5}^{1}\right)^{0,0}\right\rangle \approx-0.803 \mid 0,5\right)-0.447 \mid 1,0\right)
\end{gather*}
$$

The HCCN levels $v_{2}, v_{3}+\left(v_{4}^{1}, v_{5}^{1}\right)^{0,0} v_{3}$, and $\left(v_{4}^{1}, 3 v_{5}^{1}\right)^{0,0}$ occur at $1733.7,1759.2,1178.6$, and $1185.8 \mathrm{~cm}^{-1}$, respectively. The $v_{2}$ state of DCCN displays no resonance mixing. For $\nu_{3}$ of DCCN, we obtain

$$
\begin{gather*}
\left.\left.\left|\left(2 v_{4}^{0}, 2 v_{5}^{0}\right)^{0,0}\right\rangle \approx 0.715 \mid 0,7\right)-0.567 \mid 1,0\right)  \tag{13}\\
\left.\left.\left|v_{3}\right\rangle \approx 0.560 \mid 0,7\right)+0.720 \mid 1,0\right)
\end{gather*}
$$

where $\left(2 v_{4}^{0}, 2 v_{5}^{0}\right)^{0,0}$ and $v_{3}$ lie respectively at 1131.2 and 1150.6 $\mathrm{cm}^{-1}$.

The adiabatic projections of eqs $11-13$ are given in terms of the two dominant components, providing at least $81 \%$ of the full-dimensional wave function. For the zero-order vectors, we employ the notation used by the code, where $\mid \alpha, i)$ stands for the adiabatic level with $\alpha$ quanta in the stretching $(R)$ vibration and $i$ quanta in the five-mode vibration, as described in section 3.1.

Adiabatic projection methods are useful for quantitative studies on separability of internal molecular motions and for making quantum number assignments to the calculated fulldimensional rovibrational states. The adiabatic (zero-order) basis is, however, coordinate dependent. To verify eqs $11-13$, we studied several other orthogonal (Jacobi type) descriptions and found that the levels $v_{2}$ and $\nu_{3}$ occur as zero-order mixtures also for Jacobi type coordinates. These test calculations led additionally to the conclusion that internal motion of HCCN is most separable in the (diatom + diatom) description of Figure 1 b .
C. Transitions Involving the Bending $\boldsymbol{v}_{5}$ and $\boldsymbol{v}_{4}$ and Stretching $v_{\mathbf{1}}$ Vibrations. The transitions involving the bending $v_{5}$ and $\nu_{4}$ vibrations are summarized in Table 3, whereas Table 4 displays the transitions involving the stretching $\nu_{1}$ vibration. The level energies in Tables 3 and 4 are in excellent agreement with high-resolution spectroscopic data of refs $6,9,10$, and 12, showing a maximum deviation of $+3.3 \mathrm{~cm}^{-1}$ for the combination band $\nu_{1}+3 \nu_{5}^{3}-2 \nu_{5}^{2}$ calculated at $3464 \mathrm{~cm}^{-1}$ for HCCN. One may note that in Table 4 we also list the energies $\nu_{1}+n v_{5}^{ \pm n}-v_{1}(n=1,2,3)$, which are obtained in the semirigid bender calculations of Hung et al. ${ }^{12}$

The energy levels of the pure $\nu_{4}$ and pure $v_{5}$ bending vibrations of HCCN are graphically presented in Figure 7. The states of $\nu_{4}$ follow in particular closely the energy pattern of the two-dimensional (anharmonic) oscillator.

To quantify the degree of quasilinearity of molecular vibrations, Yamada and Winnewisser ${ }^{37}$ introduced the parameter $\gamma_{0}$, given by

$$
\begin{equation*}
\gamma_{0}=1-4\left[E\left(v_{i}^{1}\right)-E_{0}\right] /\left[E\left(2 v_{i}^{0}\right)-E_{0}\right] \tag{14}
\end{equation*}
$$

Our results from Tables 2 and 3 give for the quasilinear $\nu_{5}$ bending mode $\gamma_{0}$ of -0.12 for HCCN and of +0.05 for DCCN. On the other hand, we obtain $\gamma_{0}$ of -1.02 and -0.95 for the $\nu_{4}$ of HCCN and DCCN, respectively, as expected for a degenerate bending mode in well-behaved linear molecules. ${ }^{37}$

In Figure 7, the states with $l_{i} \neq 0$ are doubly degenerate. Pronounced $l$-type splitting is, however, found only for the $K=1$ states. For $v_{5}^{1}$ and $v_{4}^{1}$ of HCCN, the $l$-type doubling constant $q_{\mathrm{v}}$ is determined to be 40.1 and 27.6 MHz , which compare well to 40.6 and 27.9 MHz predicted by McCarthy et al. ${ }^{7}$ For the $v_{5}^{1}$ and $v_{4}^{1}$ states of DCCN, the calculated $q_{\mathrm{v}}$ values of 49.9 and 35.6 MHz agree within 0.8 MHz with the experimental data ${ }^{7}$ of 50.6 and 34.8 MHz . For the states $v_{1}+$ $v_{5}$, we obtained $q_{\mathrm{v}}$ of 42 MHz for HCCN and of 54 MHz for DCCN, which should be compared to $q_{\mathrm{v}}$ of 44 and 52 MHz estimated by Morter et al. ${ }^{6}$ and Sun et al. ${ }^{9}$

As clearly seen in Figure 7, the level separation between $v_{4} v_{4}^{\mathrm{l}_{0}}$ and $v_{4} v_{4}^{v_{4}}$ is much smaller than the separation between $v_{5} v_{5}^{l_{0}}$ and $v_{5} v_{5}^{v_{5}}$, where $l_{0}=0$ for $v_{i}$ even and $l_{0}=1$ for $v_{i}$ odd. For HCCN and DCCN, the level $2 v_{5}^{0}$, e.g., lies 118 and 108 $\mathrm{cm}^{-1}$ above $2 \nu_{5}^{2}$, respectively. An interesting difference between HCCN and DCCN is found in the ordering of the $2 v_{4}^{0}$ and $2 v_{4}^{2}$ states, Table 3. Whereas the levels $2 v_{4}^{1_{4}}$ of HCCN show the expected ordering with the level $2 \nu_{4}^{0}$ being $9 \mathrm{~cm}^{-1}$ below $2 v_{4}^{2}$, the opposite is found for DCCN. In the latter case, the level $2 v_{4}^{0}$ is $16 \mathrm{~cm}^{-1}$ above $2 v_{4}^{2}$,

The bending HCC frequency obtained for four different stretching excitations are found in Table 5. The effective frequencies $v_{5}^{\text {eff }}$ are calculated as $E_{v_{\mathrm{i}}+v_{5}}-E_{v_{\mathrm{i}}}$ using the $(J=1$, $p=1)$ values. As seen in Table 5, the excitation of $v_{1}$ and $v_{3}$ lowers $\nu_{5}^{\text {eff }}$ of HCCN approximately by 20 and $13 \mathrm{~cm}^{-1}$ and


Figure 7. Energy levels of the pure $v_{4}$ (left) and pure $v_{5}$ (right) bending vibrations of HCCN, obtained for $J=4$. One may note that the levels associated with $l_{i} \neq 0$ are doubly degenerate.


Figure 8. Effective potential energy profiles and the minimum energy path along the angle $\theta_{1}$. For the purpose of comparison, each of the curves shown is measured relative to its minimum. The minimum energy path is denoted by MEP.
$\nu_{5}^{\text {eff }}$ of DCCN by 13 and $17 \mathrm{~cm}^{-1}$, yielding changes up to $22 \%$. On the other hand, $\nu_{5}^{\text {eff }}$ increases by 12 and $16 \mathrm{~cm}^{-1}$ upon excitation of $v_{2}$ in HCCN and DCCN, respectively.

To gain a more general insight into the dynamics of the quasilinear $v_{5}$ mode, we have also carried out detailed investigations of effective (adiabatic) potential energy profiles along the angle $\theta_{1}$. For various excitations of other vibrational modes, the effective bending profiles were calculated in the spirit of the $\operatorname{DVR}(+\mathrm{R})+\mathrm{FBR}$ strategy (section 3) by solving fivedimensional eigenvalue problems for a chosen $\theta_{1}$ grid.

The effective $\theta_{1}$ profiles obtained for the five-mode ground state, $v_{1}, v_{2}$, and $v_{3}$ are depicted in Figure 8, whereas the height of the barrier to linearity, $V_{\text {eff }}^{\text {lin }}$, and the location of the minimum, $\theta_{1}^{\min }$, are presented in Table 5. As seen there, the height of the barrier to linearity on the five-mode ground-state profile is $30 \mathrm{~cm}^{-1}$ lower than the corresponding MEP value of $287 \mathrm{~cm}^{-1}$. Upon excitation of $\nu_{1}$ and $\nu_{3}$, the barrier to linearity increases by 75 and $66 \mathrm{~cm}^{-1}$ for HCCN and by 72 and $59 \mathrm{~cm}^{-1}$ for DCCN relative to the ground-state value. At the same time, $v_{2}$ excitation introduces lowering of $V_{\text {eff }}^{\mathrm{lin}}$ by $78 \mathrm{~cm}^{-1}$ for HCCN and by $81 \mathrm{~cm}^{-1}$ for DCCN. According to Table 5, the effective bending frequency $\nu_{5}^{\text {eff }}$ nicely correlates with the effective barrier height $V_{\text {eff }}^{\text {lin }}$, i.e., the higher the barrier to linearity, the lower the effective bending frequency.
D. Rotational constants. In addition to the transition energies, Tables 2-4 also give the difference $\Delta B_{\mathrm{v}}=B_{0}-B_{\mathrm{v}}$ between the ground-state rotational constant $B_{0}$ and the effective
rotational constant $B_{\mathrm{v}}$ for the vibrational state $v$. The $B_{\mathrm{v}}$ values were determined by a least-squares fit of the calculated rovibrational energies to the following approximate expression

$$
\begin{equation*}
E_{\mathrm{v}}=T_{\mathrm{v}}+B_{\mathrm{v}} J(J+1) \tag{15}
\end{equation*}
$$

where $T_{\mathrm{v}}$ stands for the term energy.
The ground-state rotational constant $B_{0}$ is calculated to be $0.364872 \mathrm{~cm}^{-1}$ for HCCN and $0.329029 \mathrm{~cm}^{-1}$ for DCCN. They are 0.0016 and $0.0014 \mathrm{~cm}^{-1}$ smaller than the experimental values of 0.366465 and $0.330441 \mathrm{~cm}^{-1}$ derived from the millimeter spectra. ${ }^{7}$ On the other hand, the theoretical results of Koput, ${ }^{21}$ which read 0.367468 and $0.331676 \mathrm{~cm}^{-1}$, overestimate $B_{0}$ by 0.0010 and $0.0012 \mathrm{~cm}^{-1}$. One may also note that the equilibrium parameters $\left(r_{1}^{\mathrm{e}}, r_{2}^{2}, r_{3}^{\mathrm{e}}, \mathrm{o}^{\mathrm{e}}, \beta^{\mathrm{e}}\right)$ obtained here (section 2.2) differ by $\left(0.0 a_{0}, 0.011 a_{0},-0.016 a_{0},-1.7^{\circ}, 0.7^{\circ}\right)$ from the best estimate of Koput ( $2.020 a_{0}, 2.510 a_{0}, 2.241 a_{0}$, $144.9^{\circ}, 175.4^{\circ}$ ) determined at the $\operatorname{CCSD}(\mathrm{T})$ level of theory.

The $\Delta B_{\mathrm{v}}$ calculated for the fundamental transitions in Table 2 agree remarkably well (within 1.5 MHz ) with the experimental values ${ }^{3,6-9,11}$ for both HCCN and DCCN. Equally good agreement is also found for $v_{5}^{1}, 2 v_{5}^{2}$, and $3 v_{5}^{3}$ of DCCN in Tables 2 and 3. In the case of HCCN, the experimental and theoretical rotational constants for $v_{5}^{1}, 2 v_{5}^{2}$, and $3 v_{5}^{3}$ exhibit, however, different trends. While the experimental rotational constants ${ }^{7}$ are nearly constant, our results in Table 3 for $2 v_{5}^{2}$ and $3 v_{5}^{3}$ of HCCN show rather prominent changes of $\Delta B_{\mathrm{v}}$ with the $l_{5}$ excitation. Similar behavior is consequently seen for the state $v_{1}+2 \nu_{5}^{2}$ in Table 4.

McCarthy et al. ${ }^{7}$ also reported three $l=0$ states at 435, 525, and $540 \mathrm{~cm}^{-1}$ for HCCN and one $l=0$ state at $311 \mathrm{~cm}^{-1}$ for DCCN, each possessing a rotational constant somewhat larger than the ground-state $B$ value, Table 3. Making use of the $\Delta B_{\mathrm{v}}$ computed in the present work, the observed $l=0$ states of HCCN are readily assigned $2 v_{5}^{0},\left(v_{4}^{1}, v_{5}^{1}\right),{ }^{0,1}$ and $\left(v_{4}^{1}, v_{5}^{1}\right),{ }^{0,0}$ while the $l=0$ state of DCCN is labeled $2 v_{5}^{0}$. For these four states, the calculated and experimental $\Delta B_{\mathrm{v}}$ results agree to better than 4 MHz in Table 3.

## 5. Summary

In this paper, we have presented a detailed study of the rovibrational energy spectra of HCCN and its isotopomer DCCN. For the ground electronic state we first developed a six-dimensional MR-ACPF potential energy surface, which is subsequently adjusted to give best agreement with experimental data for $v_{1}$ and $v_{5}$ of HCCN. Quantum mechanical calculations were carried out for total rotational angular momentum $J=$ $0-4$ using the $\operatorname{DVR}(+\mathrm{R})+\mathrm{FBR}$ method. In addition to the
transition energies, our study also includes the determination of spectroscopic parameters, such as effective rotational constants and $l$-type doubling constants.

The rovibrational energy levels of both HCCN and DCCN are studied in great detail by means of vibrationally averaged geometries and adiabatic projection methods. Resonance interaction of the $v_{2}$ stretching mode with $v_{3}+\left(v_{4}^{1}, v_{5}^{1}\right)^{0,0}$ is found in HCCN, whereas the $\nu_{3}$ stretching mode of DCCN is coupled with $\left(2 v_{4}^{0}, 2 v_{5}^{0}\right) .^{0,0}$ To gain a more general insight into the dynamics of the quasilinear $v_{5}$ bending mode, the effective (adiabatic) energy profiles along $\theta_{1}$ have been calculated for various five-mode excitations. For both HCCN and DCCN, we have found a clear increase in the height of the effective barrier to linearity for the excited $\nu_{1}$ and $\nu_{3}$, resulting in the lower effective $\nu_{5}$ bending frequency. On the other hand, $\nu_{2}$ excitation lowers the barrier to linearity and increases the effective $\nu_{5}$ frequency.

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## References and Notes

(1) Bernheim, R. A.; Kempf, R. J.; Humer, P. W.; Skell, P. S. J. Chem. Phys. 1964, 41, 1156.
(2) Wasserman, E.; Yager, W. A.; Kuck, V. J. Chem. Phys. Lett. 1970, 7, 409.
(3) Dendramis, A.; Leroi, G. E. J. Chem. Phys. 1977, 66, 4334.
(4) Saito, S.; Endo, Y.; Hirota, E. J. Chem. Phys. 1984, 80, 1427.
(5) Brown, F. X.; Saito, S.; Yamamoto, S. J. Mol. Spectrosc. 1990, 143, 203.
(6) Morter, C. L.; Farhat, S. K.; Curl, R. F. Chem. Phys. Lett. 1993, 207, 153.
(7) McCarthy, M. C.; Gottlieb, C. A.; Cooksy, A. L.; Thaddeus, P. J. Chem. Phys. 1995, 103, 7779.
(8) Miller, C. E.; Eckhoff, W. C.; Curl, R. F. J. Mol. Struct. 1995, 352/353, 435.
(9) Sun, F.; Kosterev, A.; Scott, G.; Litosh, V.; Curl, R. F. J. Chem. Phys. 1998, 109, 8851.
(10) Han, J.-X.; Hung, P. Y.; DeSain, J.; Jones, W. E.; Curl, R. F. J. Mol. Spectrosc. 1999, 198, 421.
(11) Allen, M. D.; Evenson, K. M.; Brown, J. M. J. Mol. Spectrosc. 2001, 209, 143.
(12) Hung, P. Y.; Sun, F.; Hunt, N. T.; Burns, L. A.; Curl, R. F. J. Chem. Phys. 2001, 115, 9331.
(13) Jones, W. E.; Sun, F.; Curl, R. F.; Evenson, K. M.; Brown, J. M. Can. J. Phys. 2001, 79, 389.
(14) Nimlos, M. R.; Davico, G.; Geise, C. M.; Wenthold, P. G.; Lineberger, W. C.; Blanksby, S. J.; Hadad, C. M.; Petersson, G. A.; Ellison, G. B. J. Chem. Phys. 2002, 117, 4323.
(15) Zandler, M. A.; Goddard, J. D.; Schaefer, H. F., III. J. Am. Chem. Soc. 1979, 101, 1072.
(16) Kim, K. S.; Schaefer, H. F., III; Radom, L.; Pople, J. A.; Binkley, J. S. J. Am. Chem. Soc. 1983, 105, 4148.
(17) Rice, J. E.; Schaefer, H. F., III. J. Chem. Phys. 1987, 86, 7051.
(18) Malmquist, P. A.; Lindh, R.; Roos, B. O.; Roos, S. Theo. Chim. Acta 1988, 73, 155.
(19) Seidl, E. T.; Schaefer, H. F., III. J. Chem. Phys. 1992, 96, 4449.
(20) Francisco, J. S. Chem. Phys. Lett. 1994, 230, 372.
(21) Koput, J. J. Phys. Chem. A 2002, 106, 6183.
(22) Koput, J. J. Phys. Chem. A 2003, 107, 4717.
(23) Bunker, P. R.; Landsberg, B. M. J. Mol. Spectrosc. 1977, 67, 374.
(24) Gdanitz, R. J.; Ahlrichs, R. Chem. Phys. Lett. 1988, 143, 413.
(25) Cave, R. J.; Davidson, E. R. J. Chem. Phys. 1988, 89, 6798.
(26) Szalay, P. G.; Bartlett, R. J. Chem. Phys. Lett. 1993, 214, 481.
(27) Werner, H.-J.; Knowles, P. J. Theo. Chim. Acta 1990, 78, 175.
(28) MOLPRO is a package of ab initio programs designed by H.-J. Werner and P. J. Knowles. Amos, R. D.; Bernhardsson, A.; Berning, A.; Celani, P.; Cooper, D. L.; Deegan, M. J. O.; Dobbyn, A. J.; Eckert, F.; Hampel, C.; Hetzer, G.; Knowles, P. J.; Korona, T.; Lindh, R.; Lloyd, A. W.; McNicholas, S. J.; Manby, F. R.; Meyer, W.; Mura, M. E.; Nicklass, A.; Palmieri, P.; Pitzer, R.; Rauhut, G.; Schütz, M.; Schumann, U.; Stoll, H.; Stone, A. J.; Tarroni, R.; Thorsteinsson, T.; Werner, H.-J.
(29) Meyer, W.; Botschwina, P.; Burton, P. J. Chem. Phys. 1986, 84, 891.
(30) Mladenović, M. J. Chem. Phys. 2003, 119, 11513.
(31) Mladenović, M. J. Chem. Phys. 2000, 112, 1070.
(32) Arfken, G. Mathematical Methods for Physicists, 3rd ed.; Academic Press: New York, 1985.
(33) Zare, R. N. Angular Momentum; J. Wiley \& Sons: New York, 1988.
(34) Mladenović, M. In NIC Symposium 2001; Rollnik, H., Wolf, D., Eds.; NIC Series 9; John von Neumann Institite for Computing: Jülich, Germany, 2002; pp 85-94.
(35) Mladenović, M. Spectrochim. Acta, Part A 2002, 58, 795.
(36) Echave, J.; Clary, D. C. Chem. Phys. Lett. 1992, 190, 225.
(37) Yamada, K.; Winnewisser, M. Z. Naturforsch. 1976, 31A, 139.


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    * Corresponding author. Present address: Papendiek 17, D-37073 Göttingen, Germany. E-mail: mmladen@gwdg.de.

    ث E-mail: pbotsch@gwdg.de.
    ${ }^{\text {" }}$ E-mail: cristina.puzzarini@unibo.it.

